

(3 Hours)

[Total Marks : 80]

Note:

- 1) Question No.1 is compulsory
- 2) Attempt any three out of remaining five questions
- 3) Figures to the right indicate full marks

Q1.

- a) If $\sin(\theta + i\varphi) = \tan\alpha + i\sec\alpha$, then show that $\cos 2\theta \cdot \cosh 2\varphi = 3$ [5]
- b) If $u = \log(\tan x + \tanh y)$, then show that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = 2$ [5]
- c) Express the matrix $A = \begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. [5]
- d) Expand $\sqrt{1 + \sin x}$ in ascending powers of x upto x^4 term. [5]

Q2.

- a) Find non-singular matrices P and Q such that PAQ is in normal form where, [6]

$$A = \begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix}. \text{ Also find the rank of A.}$$

- b) If $z = f(x, y)$ and $x = u \cosh v$, $y = u \sinh v$; prove that [6]

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial u}\right)^2 - \frac{1}{u^2} \left(\frac{\partial z}{\partial v}\right)^2$$

- c) Prove that $\text{Log} \left[\frac{(a-b)+i(a+b)}{(a+b)+i(a-b)} \right] = i(2n\pi + \tan^{-1} \frac{2ab}{a^2-b^2})$. Hence evaluate $\text{Log} \left(\frac{1+5i}{5+i} \right)$ [6]

Q3.

- a) If α and β are the roots of the equation $z^2 \sin^2 \theta - z \sin 2\theta + 1 = 0$, then prove that $\alpha^n + \beta^n = 2 \cos n\theta \operatorname{cosec}^{2n} \theta$ and $\alpha^n \beta^n = \operatorname{cosec}^{2n} \theta$ [6]
- b) Solve the following equations by Gauss-Seidal Method ; [6]
 $15x + 2y + z = 18$, $2x + 20y - 3z = 19$, $3x - 6y + 25z = 22$,
 Take three iterations.

- c) Prove that if z is a homogeneous function of two variables x and y of degree n , then $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n - 1)z$. Hence find the value of $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2}$ at $x = 1, y = 1$ when $z = x^6 \tan^{-1} \left(\frac{x^2 + y^2}{x^2 + xy} \right) + \frac{x^4 + y^4}{x^2 y^2}$ [8]

Q4.

- a) If $\tan (\alpha + i\beta) = \cos \theta + i \sin \theta$ then prove that $\alpha = \frac{n\pi}{2} + \frac{\pi}{4}, \beta = \frac{1}{2} \log \left(\frac{\pi}{4} + \frac{\theta}{2} \right)$ [6]
- b) Expand $x^5 + x^3 - x^2 + x - 1$ in powers of $(x - 1)$ and hence find the value of [6]
 1) $f \left(\frac{9}{10} \right)$
 2) $f(1.01)$
- c) For what values of λ and μ , the equations, [8]
 $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$
 1) have a unique solution
 2) have infinite solution

Find the solution in each case for a possible value of μ and λ .

Q5.

a) Find the nth derivative of $y = \frac{1}{x^2 + a^2}$ [6]

b) Discuss the maxima and minima of $x^3 + xy^2 - 12x^2 - 2y^2 + 21x + 16$ [6]

c) Prove that if A and B are two unitary matrices then AB is also unitary. Verify the result when

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix} \quad [8]$$

Q6.

a) If $x = \cosh\left(\frac{1}{m} \log y\right)$, prove that [6]

$$(x^2 - 1)y_{n+2} + (2n + 1)x y_{n+1} + (n^2 - m^2) y_n = 0$$

b) Find a root of the equation $xe^x = \cos x$ using the Regular Falsi Method correct to three decimal places. [6]

c) 1) Expand $\sin^4 \theta \cos^2 \theta$ in a series of multiples of θ . [4]

2) If one root of $x^4 - 6x^3 + 18x^2 - 24x + 16 = 0$ is $(1+i)$; find the other roots. [4]