

(3 hours)

[Total Marks: 80]

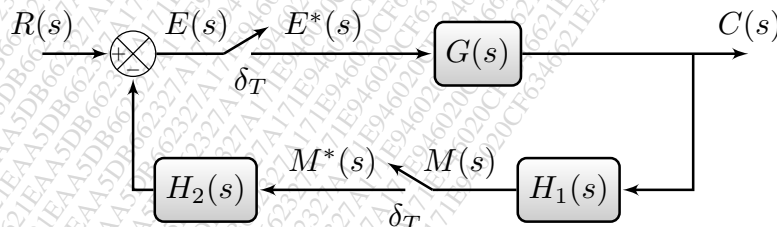
Note:

1. Question-1 is compulsory.
2. Answer any three questions from remaining five.
3. Assume suitable data if necessary.
4. Numbers in the right indicate marks.

1. Answer the following questions. (Each question carry 5 marks) 20
 - (a) Describe the data holding in digital systems and derive the transfer function of zero-order hold (ZOH).
 - (b) Draw the frequency spectrum of the ideal low-pass filter and comment on its realizability with justification.
 - (c) Explain quantization in digital control systems and derive the bound on the quantization error.
 - (d) Clearly explain the distinction between detectability and observability for discrete-time systems.

2. (a) Describe bilinear transformation approach for discretization of continuous-time systems in detail. Also, comment on the mapping between s-plane and z-plane under such discretization. 10
 - (b) Discretize continuous-time PID controller using trapezoidal rule to approximate integral term and two-point difference to approximate derivative term. 10

3. (a) Derive the closed-loop pulse transfer function $\frac{C(s)}{R(s)}$ for the digital control system shown in figure below. Assume sampling time to be 1 second. 10



- (b) Make a rough sketch of root locus of a unity feedback digital control system whose open loop transfer function is given as 10

$$G(z) = \frac{Kz(1 - e^{-T})}{(z - 1)(z - e^{-T})}$$

Also determine the critical value of K . Take sampling frequency $f_s = 1\text{Hz}$.

4. (a) Design the state feedback controller $u[k] = Kx[k] + k_0r[k]$ such that the steady-state error of the closed loop is zero for the unit step input. 10

$$x[k + 1] = \begin{bmatrix} -0.6 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & -0.8 \end{bmatrix} x[k] + \begin{bmatrix} 2 \\ 0.6 \\ 1 \end{bmatrix} u[k]$$

- (b) Represent the discrete-time system of **Q-4(a)** in controllable canonical form and observable canonical form using similarity transforms. 10
- 5. (a) Discretize the continuous time state-space equation $\dot{x} = Ax + Bu$ and obtain the discrete-time state-space representation. 10
- (b) Design a deadbeat controller for the following discrete-time system. 10

$$x[k + 1] = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} x[k] + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u[k]$$

- 6. Answer any three of the following questions. 20
 - (a) Derive the mapping between s -plane and z -plane when discretizing a continuous-time system using impulse invariance approach. Determine the region of stability on z -plane under this mapping.
 - (b) Determine the stability of a discrete-time system which has characteristic equation $z^4 + 1 = 0$.
 - (c) Explain the distinction between observability and detectability.