(3 Hours) **[Total Marks : 80]**

Note:- 1) Question number **1** is **compulsory.**

2) Attempt any **three** questions from the remaining **five** questions

3) **Figures** to the **right** indicate **full** marks.

Q.1 a) Find the Laplace transform of cost cos2t cos3t 05

b) Show that the set of functions cosnx, n= 1,2,3,……. is orthogonal over (0 , 2π) 05

c) Prove that f(z) = ($x^3 - 3xy^2 + 2xy$) + i($3x^2y - x^2 + y^2 - y^3$) is analytic and find $f^I(z)$ in terms of z . 05

d) Find the directional derivative of $\varphi = x^2+y^2+z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ at (1, 2, 3) 05

Q.2 a) Find the fourier series for f(x) = x^2 in (0 , 2π) 06

b) Show that the vector $\overline{F}$ =($x^2 + xy^2$) i + ($y^2 + x^2y$) j is irrotational and find its scalar potential 06

c) Prove that the transformation w = $\frac{1}{z+i}$ transforms real axis of z- plane into a circle of w – plane 08

Q.3 a) Using convolution theorem, find inverse Laplace transform of $\frac{s^2}{(s^2+2^2)^2}$. 06

b) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right)$ 06

c) Find half range cosine series for f(x) = $x(\pi - x)$, 0<x< π. Hence show that $\sum_1^\infty \frac{1}{n^4} = \frac{\pi^4}{90}$ 08

Q.4 a) Evaluate by Green's theorem $\int_c (e^{x^2} - xy)\,dx - (y^2 - ax)dy$ where c is the circle $x^2 + y^2 = a^2$. 06

b) Prove that $2\,J_0''(x) = J_2(X) - J_0(x)$. 06

c) i) Evaluate $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t}$ dt 08

ii) Find Laplace transform of $t\sqrt{1 + sint}$

Q.5 a) Find the orthogonal trajectory of the family of curves $x^3y - xy^3 = c$. 06

b) Prove that $\int x \,.\, J_{2/3}\ (x^{3/2})\ dx = -\ \frac{2}{3} x^{-1/2} J_{-1/3}\ (x^{3/2})$. 06

c) Obtain complex form of Fourier Series for $f(x) = e^{2x}$ in (0, 2). 08

Q.6 a) Use stoke's Theorem to evaluate $\int_c \bar{F} \,.\, d\bar{r}$ where $\bar{F}$ = yz i + zx j + xy k and C is the boundary of the circle $x^2 + y^2 + z^2 = 1$ and $z = 0$. 06

b) Find the fourier integral representation for 06

$f(x) = e^{ax}$, $x \le 0$, $a > 0$

$= e^{-ax}$, $x \ge 0$, $a > 0$

Hence show that $\int_0^\infty \frac{\cos wx}{w^2 + a^2}\, dx = \frac{\pi}{2a} e^{-ax}$, $x > 0$, $a > 0$

c) Solve using Laplace transform $(D^2 + 2D + 5)y = e^{-t}sint$, where $y(0) = 0$, $y'(0) = 1$. 08

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