[Time: 3 Hours]   **[Marks:80]**

Please check whether you have got the right question paper.

N.B: 1.   Question 1 is compulsory.
       2.   Attempt any three questions from Q.2 to Q.6.

Q1 (a)   Prove $cosh^5x = \cosh5x + 5\cosh3x + 10\cosh x$ **(3)**

(b)   If u= log(tanx+ tany)   **(3)**
       Prove $\sin2x \frac{\partial u}{\partial x} + \sin2y \frac{\partial u}{\partial y} = 2$

(c)   **(3)**
       If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$

(d)   Express the following matrix as sum of symmetric and skew symmetric   **(3)**
      matrix.
$$A= \begin{pmatrix} 2 & 2+i & 3 \\ -2+i & 0 & 4i \\ -i & 3-i & 1-i \end{pmatrix}$$

(e)   Show that $\log(1+ \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6}$ **(4)**

(f)   If $y = \frac{x^2}{(x-1)(x-2)}$ Find y_n **(4)**

Q2 (a)   Solve the Equation $x^4 - x^3 + x^2 - x + 1 = 0$ **(6)**

(b)   Reduce the following Matrix to the Normal form and hence find the   **(6)**
      rank of the matrix
$$A= \begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$$

(c)   If $u = \frac{x^2y^2z^2}{x^2+y^2+z^2} + \cos^{-1}\left(\frac{xy+yz}{\sqrt{x^2+y^2+z^2}}\right)$ **(8)**

Find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$

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Q3 (a)   (a) Investigate for what values of λ and μ the system of equations   **(6)**
     x+2y+3z= 4, x+3y+4z = 5, x+3y+λz =μ.
     have 1) unique solution, 2) Infinite solutions, 3) No solution

(b)   Find the Extreme values of f(x,y)= xy+ $a^3(\frac{1}{x} + \frac{1}{y})$ **(6)**

(c)   Separate into real and imaginary parts of $\tan^{-1}(e^{i\theta})$ **(8)**

Q4 (a)   If $u^2 + xv^2 = x + y$, $v^2 + yu^2 = x - y$ Find $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}$ **(6)**

(b)   If log cos(x+iy)= a+ib   Prove $2e^{2a}= $ cosh2y +cos 2x   **(6)**

(c)   **(8)**
     Solve the following Equations by Gauss Seidel method Upto four
     iterations.
         4x-2y-z= 40 , x-6y+2z= -28 , x-2y+12z= -86

Q5 (a)   Using De Moivre's theorem Prove   **(6)**
   $cos^7\theta = \frac{1}{2^6}(\cos 7\theta + 7 \cos 5\theta + 21 \cos 3\theta + 35 \cos \theta)$

(b)   Evaluate $\lim\limits_{x\to 0} \left(\frac{1}{x^2} - cot^2x\right)$ **(6)**

(c)   If $y = \sin(m \sin^{-1} x)$ Prove that $(1-x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$ **(8)**
              And hence find $y_3(0)$.

Q6 (a)   (a) Show the following vectors are linearly dependent and find the   **(6)**
     relation between them.
     $[2, -1,3,2]$,$[1,3,4,2]$,$[3, -5,2,2]$.

(b)   If z=f(x,y) where x = u cosh v ,y = usinh v Prove   **(6)**
     $(\frac{\partial z}{\partial x})^2 - (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial u})^2 - \frac{1}{u^2}(\frac{\partial z}{\partial v})^2$

(c)   Fit the curve of the form y= ab^x to the following data.   **(8)**

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.5

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