Q. P. Code: 24492

Duration: 3 Hours **Total Marks: 80**

N.B. : 1) Q.1. is compulsory.

2) Attempt any three from the remaining.

Q.1. a) Show that the set $\{e^x, xe^x, x^2e^x\}$ is linearly independent in $C^2(-\infty, \infty)$. (5)

b) Show that $\int_C \log z dz = 2\pi i$, where C is the unit circle in the z-plane. (5)

c) Find the projection of u=(3,1,3) along and perpendicular to v=(4,-2,2) (5)

d) Find the extremal of $\int_{x_1}^{x_2} \left(y^2 + y'^2 + 2ye^x \right) dx$ (5)

Q.2. a) If $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$, find e^A (6)

b) Evaluate $\int_0^{\pi} \frac{d\theta}{3 + 2\cos\theta}$ (6)

c) Find the singular value decomposition of $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ (8)

Q.3. a) Find the extremal of $\int_0^{\pi} \left(y'^2 - y^2 \right) dx$ given $y(0) = 0, y(\pi) = 0$ (6)

b) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and hence find A^{-1} & A^4 (6)

c) Expand $f(x) = \frac{1}{(z-1)(z-2)}$ in the regions (i) $1 < |z-1| < 2$ (ii) $|z| < 1$ (8)

Q.4. a) Construct an orthonormal basis of R^3 using Gram Schmidt process to
S = {(3,1),(2,3)} (6)

b) Find the extremum of $\int_{x_0}^{x_1} (2xy + y''^2) dx$. (6)

c) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy + 4xz - 2zy$ to canonical form and hence, find its rank, index and signature and value class. (8)

Q.5. a) Using Residue theorem evaluate $\int_C \frac{z^2}{(z-1)^2(z+1)}dz$ where C is |z|=2. (6)

b) Find the linear transformation Y=AX which carries $X_1 = (1,0,1)', X_2 = (1,-1,1)', X_3 = (1,2,-1)'$ onto $Y_1 = (2,3,-1)', Y_2 = (3,0,-2)', Y_3 = (-2,7,1)'$ (6)

c) Check whether $V = \mathbb{R}^2$ is a vector space with respect to the operations

$(x_1,0)+(x_2,0) = (x_1+x_2,0); k(x_1,0) = (kx_1,0)$ (8)

Q.6.a) Obtain Taylor's series expansion for $f(x) = \frac{2z^3+1}{z(z+1)}$ about $z = i$ (6)

b) Let $W = span\left\{(0,1,0), \left(\frac{-4}{5},0,\frac{3}{5}\right)\right\}$, Express w =(1,2,3) in the form of $w = w_1 + w_2$ where

$w_1 \in W$ & $w_2 \in W^{\perp}$ (6)

c) Using Rayleigh- Ritz method, solve the boundary value problem $I = \int_0^1 \left(2xy - y^2 - y'^2\right)dx$;

given $y(0) = y(1) = 0$ (8)

****