(Time: 3 hours)   **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.
  (2) Answer any three questions from Q.2 to Q.6.
  (3) Use of Statistical Tables permitted.
  (4) Figures to the right indicate full marks.

Q.1 (a) Find all basic, feasible and degenerate solutions for the following equations:
$2x_1 + 6x_2 + 2x_3 + x_4 = 3$; $6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$ **05**

(b) Integrate the function $f(z) = x^2 + i\,xy$ from A (1,1) to B (2,4) along the curve
$x = t, y = t^2$.   **05**

(c) A machine is set to produce metal plates of thickness 1.5 cms with S.D. of 0.2 cms.
A sample of 100 plates produced by the machine gave an average thickness of 1.52   **05**
cms. Is the machine fulfilling the purpose? Test at 1% Level of Significance.

(d) The sum of the Eigen values of a 3×3 matrix is 6 and the product of the Eigen values
is also 6. If one of the Eigen value is one, find the other two Eigen values.   **05**

Q.2 (a) Evaluate $\oint \frac{sin^6 z}{(z - \pi/6)^n} dz$ where c is the circle $|z| = 1$ for n = 1, n =3.   **06**

(b) Solve the following LPP using Simplex Method
  Maximize $z = 3\,x_1 + 5x_2$
   subject to
$$3x_1 + 2x_2 \le 18,$$
$$x_1 \le 4 ,$$
$$x_2 \le 6 ,$$
$$x_1 , x_2 \ge 0$$
                                                                 06

(c) The following data is collected on two characters. Based on this, can you say that
there is no relation between smoking and literacy? Use Chi-square test at 5% Level
of significance.   **08**

	Smokers	Non-smokers
Literates	40	35
Iliterates	35	85

Q.3 (a) Find the Eigen values and Eigen vectors of the following matrix.   **06**
$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

(b) The incomes of a group of 10,000 persons were found to be normally distributed with
mean of rs. 750 and Standard deviation of rs. 50. What is the lowest income of richest   **06**
250?

(c) Obtain Taylor's and Laurent's expansions of $f(z) = \frac{z-1}{z^2 - 2z - 3}$ indicating region
of convergence.   **08**

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Q.4 **(a)** A man buys 100 electric bulbs of each of two well-known makes taken at random from stock for testing purpose. He finds that 'make A' has a mean life of 1300 hrs with a S.D. of 82 hours and 'make B' has a mean life of 1248 hours with S.D. of 93 hours. Discuss the significance of these results. **06**

(b) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$. **06**

(c) (i) Out of 1000 families with 4 children each, how many would you expect to have (a) at least one boy (b) at most 2 girls.

(ii) Find the Moment Generating Function of Binomial Distribution and hence find its mean. **04+04**

Q.5 **(a)** Check whether the following matrix is Derogatory or Non-Derogatory:
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$
06

(b) The means of two random samples of sizes 9 and 7 are 196.42 and 198.82 respectively. The sum of the squares of the deviations from the means are 26.94 and 18.73 respectively. Can the samples be regarded to have been drawn from the same normal population? **06**

(c) Use the dual simplex method to solve the following L.P.P.

Minimise $z = x_1 + x_2$

subject to
$$2x_1 + x_2 \geq 2$$
$$-x_1 - x_2 \geq 1$$
$$x_1, x_2 \geq 0$$
08

Q.6 **(a)** Show that the matrix A satisfies Cayley-Hamilton theorem and hence find A^{-1}.

Where $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$
06

(b) A random variable X has the probability distribution
$$P(X = x) = \frac{1}{8}\ 3c_x, x = 0, 1\ 2, 3. \quad \text{Find mean and variance.}$$
06

(c) Using Kuhn-Tucker conditions, solve the following NLPP

Maximize $z = 10x_1 + 10x_2 - x_1^2 - x_2^2$

subject to
$$x_1 + x_2 \leq 8$$
$$-x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0$$
08