(3 hours)   Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.
  (2) Answer any three questions from Q.2 to Q.6.
  (3) Use of Statistical Tables permitted.
  (4) Figures to the right indicate full marks.

Q.1 **(a)** Find all the basic solutions to the following problem:

$Maximise \quad z = x_1 + 3x_2 + 3x_3$

$subject \ to \quad x_1 + 2x_2 + 3x_3 = 4$ 05

$\qquad\qquad\qquad 2x_1 + 3x_2 + 5x_3 = 7$

$\qquad\qquad\qquad x_1, x_2, x_3 \geq 0$

(b) Evaluate $\int_c \left(z - z^2\right) dz$, where c is upper half of the circle $|z| = 1$.   05

(c) Ten individual are chosen at random from a population & heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71.inches. Discuss the suggestion that the height of universe is 65 inches.   05

(d) If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$, find A^{100} 05

Q.2 **(a)** Evaluate $\int_c \dfrac{z+2}{(z-3)(z-4)} \ dz$, where c is the circle $|z| = 1$ 06

(b) An I.Q. test was administered to 5 persons and after they were trained. The results are given below.

	I	II	III	IV	V
I.Q. Before training	110	120	123	132	125
I.Q. after training	120	118	125	136	121

06

Test whether there is any change in I.Q. after the training programme, use 1% LOS.

(c) Solve the following LPP using Simplex Method

$Maximise \quad z = 4x_1 + 10x_2$

$subject \ to \quad 2x_1 + x_2 \leq 10$

$\qquad\qquad\quad 2x_1 + 5x_2 \leq 20$ 08

$\qquad\qquad\quad 2x_1 + 3x_2 \leq 18$

$\qquad\qquad\quad x_1, x_2 \geq 0$

Q.3 **(a)** Find the Eigen values and Eigen vectors of the following matrix.

$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ 06

[Turn over

(b) If the height of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches. Find the expected number of students having heights between 65 & 71 inches. **06**

(c) Obtain Taylor's and Laurent's expansions of $f(z) = \dfrac{z^2 - 1}{z^2 + 5z + 6}$ around $z = 0$ **08**

Q.4 (a) A machine is claimed to produce nails of mean length 5 cms & standard of 0.45 cm. A random sample of 100 nails gave 5.1 as their average length. Does the performance of the machine justify the claim? Mention the level of significance you apply. **06**

(b) Using the Residue theorem, Evaluate $\displaystyle\int_0^{2\pi} \dfrac{d\theta}{5 + 3\sin\theta}$ **06**

(c) (i) In a certain manufacturing process 5% of the tools produced turnout to be defective. Find the probability that in a sample of 40 tools at most 2 will be defective.

(ii) A random variable x has the probability distribution **04+04**

$P(X = x_i) = \dfrac{1}{8}{}^3C_x$, $X = 0,1,2,3$. Find the moment generating function of x

Q.5 (a) Check whether the following matrix is Derogatory or Non-Derogatory:

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **06**

(b) In an industry 200 workers employed for a specific job were classified according to their performance & training received to test independence of training received & performance. The data are summarized as follows. **06**

Performance	Good	Not good	Total
Trained	100	50	150
Untrained	20	30	50
Total	120	80	200

Use χ^2-test for independence at 5% level of significance & write your conclusion.

(c) Use the dual simplex method to solve the following L.P.P.

Minimise $\quad z = 2x_1 + x_2$

subject to $\quad 3x_1 + x_2 \geq 3$

$\qquad\qquad 4x_1 + 3x_2 \geq 6$

$\qquad\qquad x_1 + 2x_2 \leq 3$

$\qquad\qquad x_1, x_2 \geq 0$ **08**

[Turn over

Q.6 (a) Show that the matrix A satisfies Cayley-Hamilton theorem and hence find A^{-1}.

Where $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

06

(b) A discrete random variable has the probability density function given below

$X = x_i$	-2	-1	0	1	2	3
$P(x_i)$	0.2	K	0.1	2K	0.1	2K

Find K, Mean, Variance.

06

(c) Using Kuhn-Tucker conditions, solve the following NLPP

$Maximise \quad z = 2x_1^2 - 7x_2^2 + 12x_1 x_2$

$subject \ to \quad 2x_1 + 5x_2 \leq 98$

$\qquad\qquad x_1, x_2 \geq 0$

08

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