(3 Hours) **(Total Marks : 80)**

N.B.: 1) **Question No. 1** is **Compulsory**.
2) Attempt **any three** from the **remaining**.

1. **a)** Find the extremal of $\int_{x_0}^{x_1} \frac{1+y^2}{y'^2} dx$. **(05)**

b) Is the following set of vectors in P_2 linearly independent? $2 - x + 4x^2$, $3 + 6x + 2x^2,\ 2 + 10x - 4x^2$? **(05)**

c) Show that Eigen values of Hermitian matrix are real. **(05)**

d) Evaluate $\int (z^2 - 2\bar{z} + 1)\, dz$ over a closed circle $x^2 + y^2 = 2$. **(05)**

2. **a)** Find the extremal $\int_0^{\pi} \left(y^2 - y'^2 - 2y cos x\right) dx,\quad y(0) = 0,\ y(\pi/2) = 0$. **(06)**

b) Find the Eigen Values and Eigen Vectors of the matrix $A^3 + 3I$, where **(06)**

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

c) Obtain all possible expansion of $f(z) = \frac{z}{(z-1)(z-2)}$ about $z = -2$ indicating region of convergence. **(08)**

3. **a)** Verify Cayley - Hamilton Theorem for $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$ and find A^{-1}. **(06)**

b) Using Cauchy's Residue Theorem evaluate $\int_C \frac{e^z}{z^2 + \pi^2} dz$ where C is |z|=4. **(06)**

c) Show that a closed curve 'C' of a given fixed length (perimeter) which encloses maximum area is a circle. **(08)**

4. **a)** Find an orthonormal basis for the subspace of R^3 by applying Gram-Schmidt process, where $u_1 = (1,0,1,1), u_2 = (-1,0,1,1), u_3 = (0,-1,1,1)$. **(06)**

b) Find A^{20} for the matrix $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$. **(06)**

c) Reduce the Quadratic Form $2xy + 2yz + 2zx$ to diagonal form by orthogonal reduction method. **(08)**

5. **a)** Using Rayleigh-Ritz Method, find an approximate solution to the extremal problem $\int_0^1 \left(y'^2 - y^2 - 2yx\right) dx,\quad y(0) = 0,\ y(1) = 0$. **(06)**

b) Let V be a vector space containing 2×2 matrices and $W \subseteq V$ such that $W = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. Is W a subspace of V? Justify. **(06)**

c) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diogonable.Also find the transforming matrix and diagonal matrix. **(08)**

6. **a)** Using Cauchy's Residue Theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\, sin\theta}$. **(06)**

b) Evaluate $\int_{1-i}^{2+i} (2x + 1 + iy) dz$ along the curve $x = t + 1, y = 2t^2 - 1$. **(06)**

c) Find the singular value decomposition of the matrix A= $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ **(08)**
