

Duration: 3 Hours

Marks: 80

N.B: a) Question number 1 is compulsory

b) Solve any three from the remaining.

c) All the question carry equal marks

1. a) Find the extremal of  $\int_0^\pi \frac{1+y^2}{y'^2} dx$  subject to  $y(0) = 0, y(\pi) = 0$ . [5]

b) Using Cauchy's Schwartz Inequality, show that  $(a\cos\theta + b\sin\theta)^2 \leq a^2 + b^2$ ,

Where 'a' and 'b' are real. [5]

c) Show that Eigen values of Hermitian matrix are real. [5]

d) Evaluate  $\int (z^2 - 2\bar{z} + 1) dz$  over a closed circle  $x^2 + y^2 = 2$ . [5]

2. a) Find the extremal  $\int_{x_1}^{x_2} (y^2 - y'^2 - 2ycoshx) dx$  [6]

b) Find the Eigen values and Eigen Vectors of the matrix  $A^2 + 3I$ , where [6]

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

c) Obtain all possible expansion of  $f(z) = \frac{1}{z^2(z-1)(z+2)}$  about  $z = 0$  indicating region of convergence. [8]

3. a) Verify Cayley - Hamilton Theorem for  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$  and find  $A^{-1}$ . [6]

b) Using Residue theorem evaluate  $\int_C \frac{e^z}{z^2 + \pi^2} dz$  where C is  $|z|=4$ . [6]

c) Show that a closed curve 'C' of a given fixed length (perimeter) which encloses maximum area is a circle. [8]

4. a) Find an orthonormal basis for the subspace of  $R^3$  by applying Gram-Schmidt process, where  $u_1 = (1,0,0), u_2 = (3,7,-2), u_3 = (0,4,1)$ . [6]

b) Find  $A^{50}$  for the matrix  $A = \begin{bmatrix} 4 & 3 \\ 7 & 8 \end{bmatrix}$  [6]

- c) Reduce the Quadratic Form  $xy + yz + zx$  to normal form by congruent transformation. [8]
5. a) Using Rayleigh-Ritz Method, find an approximate solution to the extremal problem  $\int_0^1 (y^2 + 2yx - y'^2) dx$ ,  $y(0) = 0$ ,  $y(1) = 0$ . [6]
- b) Determine whether the set  $V = \{(x, y, z) : x = 1, y = 0 \text{ or } z = 0\}$  is a subspace of  $R^3$  [6]
- c) Show that the matrix  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$  is diagonalizable. Also find the transforming matrix and diagonal matrix. [8]
6. a) Using Cauchy's Residue Theorem, evaluate  $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$  [6]
- b) Evaluate  $\int_{1-i}^{2+i} (2x + 1 + iy) dz$  along the straight line joining  $A(1, -1)$  and  $B(2, 1)$  [6]
- c) Find the singular value decomposition of the matrix  $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$  [8]

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