

Q. P. Code: 37525

(3 hours)

Total marks: 80

N.B.: (1) Question No. 1 is compulsory

(2) Attempt any Three from remaining

Q1 a) If X_1 has mean 4 and variance 9 & X_2 has mean -2 and variance 4 [5]
where X_1 & X_2 are independent, find $E(2X_1 + X_2 - 3)$ and
 $V(2X_1 + X_2 - 3)$.

b) Find the extremals of $\int_{x_1}^{x_2} (x + y')y' dx$ [5]

c) Verify Cauchy Schwartz inequality for the vectors $u = (-4, 2, 1)$ and [5]
 $v = (8, -4, -2)$

d) Check whether $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is derogatory or not. [5]

Q2 a) Using Cauchy's Residue theorem evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}$ where C is [6]
 $|z| = 4$

b) Show that the extremal of the isoperimetric problem [6]

$I[y(x)] = \int_{x_1}^{x_2} (y')^2 dx$ subject to the condition $\int_{x_1}^{x_2} y dx = k$ is a
parabola.

c) Is the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalisable? If so find the diagonal [8]
matrix and the transforming matrix.

Q3 a) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ [6]
hence find A^{-1}

b) Check whether the following are subspaces of \mathbb{R}^3 [6]

(i) $W = \{(a, 0, 0) \mid a \in \mathbb{R}\}$

(ii) $W = \{(x, y, z) \mid x = 1, z = 1, y \in \mathbb{R}\}$

c) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in Taylor's & Laurent's series indicating [8]
regions of convergence.

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Q4 a) Using Rayleigh-Ritz method to solve the boundary value problem [6]

$$I = \int_0^1 (2xy + y^2 - (y')^2) dx ; 0 \leq x \leq 1 \text{ given } y(0) = y(1) = 0$$

b) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that $3 \tan A = A \tan 3$. [6]

c) If sizes of 10,000 items are normally distributed with mean 20 cms & standard deviation of 4 cms. Find the probability that an item selected at random will have size : [8]

(i) between 18 cms and 23 cms , (ii) above 26 cms

Q5 a) Find orthonormal basis of \mathbb{R}^3 using Gram-Schmidt process where [6]

$$S = \{(1,0,0), (3,7,-2), (0,4,1)\}$$

b) In a factory, machines A , B & C produce 30%, 50% & 20% of the total production of an item. Out of their production 80% , 50% & 10% are defective respectively. An item is chosen at random and found to be defective. What is the probability that it was produced by machine A. [6]

c) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$ [8]

Q6 a) Evaluate $\int_C \frac{dz}{z^3(z+4)}$ where C is the circle [6]

(i) $|z| = 2$ and (ii) $|z - 3| = 2$

b) Two unbiased dice are thrown three times, using Binomial distribution find the probability that the sum nine would be obtained (i) once , (ii) twice [6]

c) For the following data [8]

| | | | | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| X | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 |
| Y | 45 | 51 | 54 | 61 | 66 | 70 | 74 | 78 | 85 | 89 |

Find the coefficients of regression b_{xy} & b_{yx} and the coefficient of correlation (r)
