

Duration : 3 Hours

Marks: 80

N.B .1) Question No. 1 is compulsory .

2) Attempt any three questions out of the remaining five questions .

3) Figures to the right indicate full marks .

- Q.1** a) Find the Laplace transform of $f(t) = 2\alpha(1 + te^{-t})^2 . e^{-2t}$ where α is real constant. 5
- b) Find the Fourier series for $f(x) = x$ in $(-3,3)$ 5
- c) In what direction is the directional derivative of $\phi(x, y, z) = 2x^2y^2(8z^4)$ at $(1,-1,-2)$ is maximum ? Find its magnitude . 5
- d) Determined the constants A,B,C,D,E & F if $f(z) = (Ax^3 - Bxy^2 + \sin 6x . \cosh 6y + C . x) + i(Dyx^2 - 9y^3 + \cos Ex \sinh Fy + 101y)$ is analytic . 5
- Q.2** a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. 6
- b) Evaluate $\int_0^\infty e^{-8t} \left\{ \int_0^t \int_0^t \int_0^t x . \sin 4x . \cos 4x . (dx)^3 \right\} . dt$ 6
- c) Obtain half range cosine series for $f(x) = x$, $0 < x < 1$ and hence prove that the value of $\frac{\pi^4}{96} = \sum_{n=1}^\infty \frac{1}{(2n-1)^4}$ using Parseval's identity . 8
- Q.3** a) If $\vec{F} = (x + 2y + 2Lz)i + (4Mx - 3y - z)j + (4x + Ny + 2z)k$ is irrotational .Find the constants L, M, N .Show that \vec{F} can be expressed as the gradient of the scalar function . 6
- b) Find Fourier series for the following function 6
- $$f(x) = \begin{cases} (x - \pi)^2 & 0 \leq x \leq \pi \\ 0 & \pi \leq x \leq 2\pi \end{cases}$$
- c) Solve using Laplace transform $(D^2 + 25)y = (K + 6) . t$,if $y(0) = 0$, $y'(0) = 0$ and find the value of the constant K if $y(\pi) = 1$. 8
- Q.4** a) Find the translation transformation using cross ratio property, which maps the points $\infty, -1, 1$ of Z-plane onto the points $\infty, 3, 2$ of W-plane . 6

b) By using Stokes theorem evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 + 2y^2)i + (2x^2 - y^2)j$ and C is the boundary of the region enclosed by circle $x^2 + y^2 = 9$, $x^2 + y^2 = 36$. 6

c) Find Inverse Laplace transform of
 i) $\left\{ \frac{s+\alpha}{s^2+16} \right\}$ and find the constant α , if $f\left(\frac{\pi}{8}\right) = 1$ 8
 ii) $\left\{ \frac{s+6}{(s-5)^2+121} \right\}$

Q.5 a) Define Orthogonal set of functions on (a,b). If $f(x) = P_1f_1(x) + P_2f_2(x) + P_3f_3(x)$, where P_1, P_2, P_3 are constants and $f_1(x), f_2(x), f_3(x)$ are orthogonal functions on (a,b), Then show that $\int_a^b [f(x)]^2 \cdot dx = P_1^2R_1 + P_2^2R_2 + P_3^2R_3$ where R_i ' are non zero for $i = 1,2,3$. 6

b) Find the analytic function $f(z) = u + iv$ in terms of Z if $u - 3v = x^2 - y^2 - 5x + y + 2$. 6

c) Verify Green's theorem for $\int_C (x^2)dx - (xy)dy$, C is a triangle whose vertices are A(0,2), B(2,0), C(4,2) in the XY-plane. 8

Q.6 a) Find the image of the real axis of the Z-plane under the transformation $w = \frac{1}{z+i}$ onto the W-plane. 6

b) Find Laplace transform of $f(t) = e^{-4t} \cdot \cos 4t \cdot \sin 4t \cdot H\left(t - \frac{\pi}{2}\right)$ 6

c) Obtain Complex form of Fourier series for $f(x) = \cosh 2x + \sinh 2x$ in $(-2,2)$. 8
