

Duration – 3 Hours

Total Marks : 80

N.B.:- 1. Question no 1 is compulsory.

2. Attempt any THREE questions out of remaining FIVE questions.

- Q.1 a) Find Laplace Transform of the given function $f(t) = e^{-5t} \operatorname{erf}(\sqrt{t})$. (5)
- b) Prove that $J_{-\frac{3}{2}} = -\sqrt{\frac{2}{\pi x}} \left(\frac{\cos x}{x} + \sin x \right)$ (5)
- c) Find complex form of Fourier series of $\cosh 2x + \sinh 2x$; $(-\pi, \pi)$. (5)
- d) Find the Directional derivative of $F = x^2 - y^2 + 2z^2$ at $P(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$. In what direction will it be maximum? What is the magnitude of this maximum? (5)
- Q.2 a) Prove that $\nabla \times \left[\frac{\vec{a} \times \vec{r}}{r^3} \right] = \frac{-\vec{a}}{r^3} + \frac{3(\vec{a} \cdot \vec{r})\vec{r}}{r^5}$. (6)
- b) Show that the set of functions $\left\{ \sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), \dots \right\}$ forms an orthogonal set over the interval $[0, L]$. Construct corresponding orthonormal set. (6)
- c) Determine the analytic function $f(z) = u + iv$ if $3u + 2v = y^2 - x^2 + 16xy$. (8)
- Q.3 a) Find the Bilinear transformation that maps the points $z = 1, i, -1$ into $w = 0, 1, \infty$. (6)
- b) Prove that $\int_0^{\infty} \frac{e^{-\sqrt{2}t} \sin t \sinh t}{t} dt = \frac{\pi}{8}$ (6)
- c) Obtain Fourier series of $f(x) = \begin{cases} x + \pi/2 & -\pi \leq x \leq 0 \\ \pi/2 - x & 0 \leq x \leq \pi \end{cases}$ (8)
- Hence deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$
- Q.4 a) Find the Fourier sine transform of e^{-x} , $x \geq 0$, and hence deduce that $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$, $m \geq 0$. (6)
- b) Find Inverse Laplace Transform of $\frac{(s+2)^2}{(s^2+4s+8)^2}$ using Convolution theorem. (6)
- c) Verify Green's theorem for $\vec{F} = (x^2 - xy)i + (x^2 - y^2)j$ and C is the closed curve formed by $x^2 = 2y, x = y$. (8)

Q.5 a) Prove that $\int J_5(x)dx = -J_4 - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x)$. (6)

b) Evaluate $\iint_S \vec{F} \cdot \vec{n} dS$ where S is the surface of the region bounded by (6)

$x^2 + y^2 = 4, z = 0, z = 3$ and $\vec{F} = 4xi - 2y^2j + z^2k$.

Find inverse Laplace transform of

c) (i) $\log\left(\frac{s^2 + a^2}{(s+b)^2}\right)$ (4)

(ii) $\frac{e^{-2s}}{s^2 + 8s + 25}$ (4)

Q.6 a) Prove that $\vec{F} = (6xy^2 - 2z^3)i + (6x^2y + 2yz)j + (y^2 - 6z^2x)k$ is a conservative (6)
field. Find the scalar potential for \vec{F} . Hence find the work done in moving a
particle from (1,0,2) to (0,1,1)

b) Express the function $f(x) = -e^{-kx}$, for $x < 0$ & $f(x) = e^{-kx}$, for $x > 0$ as (6)
Fourier integral and hence deduce that

$\int_0^\infty \frac{\omega \sin \omega x}{\omega^2 + k^2} d\omega = \frac{\pi}{2} e^{-kx}$, if $x > 0, k > 0$.

c) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}$, $y(0) = 4$, $y'(0) = 2$ using Laplace Transform. (8)
