Q.P. Code: 37608

Duration – 3 Hours **Total Marks : 80**

(1) N.B.:-Question no 1 is compulsory.

(2) **Attempt any THREE questions out of remaining FIVE questions.**

Q.1) a) Find Laplace Transform of the periodic function $|\sin t|$. (5)

b) Find the half range sine series of $f(x) = lx - x^2$, in $(0, l)$. (5)

c) Find the directional derivative of $x^3 + y^3 + z^3 - xyz$ at P(1,1,1) in the direction normal to the surface $x \log z + y^2 = 4$ at Q(1,-2,1). (5)

d) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. (5)

Q.2 a) Show that the image of the rectangular hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$ is the lemniscates $\rho^2 = \cos 2\phi$. (6)

b) Evaluate $\int_0^\infty e^{-t}\left(\int_0^t u^4 \sinh u \cosh u \, du\right) dt$. (6)

c) Obtain Fourier series of $f(x) = \begin{cases} 1 + (2x/\pi) & -\pi \le x \le 0 \\ 1 - (2x/\pi) & 0 \le x \le \pi \end{cases}$. Hence deduce that (8)

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

Q.3 a) Show that the set of functions $\left\{ Sin\left(\frac{\pi x}{2L}\right), Sin\left(\frac{3\pi x}{2L}\right), Sin\left(\frac{5\pi x}{2L}\right), \dots\dots \right\}$ forms an Orthogonal set over the interval $[0, L]$. Construct corresponding orthonormal set. (6)

b) Prove that $\nabla \times \left[\frac{\bar{a} \times \bar{r}}{r^3}\right] = \frac{-\bar{a}}{r^3} + \frac{3(\bar{a} \bullet \bar{r})\,\bar{r}}{r^5}$. (6)

c) Find the analytic function f(z) = u + iv ,if $u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$. (8)

Q. 4 a) Verify Green's theorem for $\int_C (xy + y^2)dx + x^2 dy$ where C is the closed path formed by $y = x, y = x^2$. (6)

b) Prove that $\int J_5(x)dx = -J_4 - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x)$. (6)

c) Solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=3te^{-t}$, $y(0)=4$, $y'(0)=2$ using Laplace Transform. **(8)**

Q. 5 a) Find the Bilinear Transformation which maps z= 1, i, -1 onto the points w= i,0,-i. (6)

b) Find Inverse Laplace Transform of $\frac{(s+2)^2}{(s^2+4s+8)^2}$ using Convolution theorem. (6)

c) Evaluate $\iint_S \overline{F}\bullet \overline{n}dS$ where S is the surface of the cube bounded by $x=0, y=0, z=0, x=1, y=1, z=1$ and $\overline{F}=4xzi-y^2j+yzk$. **(8)**

Q. 6 a) Evaluate $\int_C (x+2y)dx+(x-z)dy+(y-z)dz$ where C is the boundary of the triangle with vertices $(2,0,0),(0,3,0),(0,0,6)$ oriented in the anti-clockwise direction. (6)

b) Find the Fourier integral representation for $f(x)=\begin{Bmatrix}1-x^2 \; for \; |x|\le 1\\ 0 \quad for \quad |x|>1\end{Bmatrix}$ (6)

c) Evaluate the inversr Laplace transformation of **(8)**

a) $\log\left(\frac{s^2+a^2}{(s+b)^2}\right)$ b) $\frac{e^{-2s}}{s^2+8s+25}$.

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