

(3 Hours)

[Total marks : 80

- Note** :-
- 1) Question number 1 is **compulsory**.
 - 2) Attempt any **three** questions from the remaining **five** questions.
 - 3) **Figures** to the **right** indicate **full marks**.

- Q.1
- a) Find the Laplace transform of $\sinh^5 t$. 05
 - b) Find an analytic function whose imaginary part is $e^{-x}(y \cos y - x \sin y)$. 05
 - c) Find the Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$. 05
 - d) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 2x i + (xz - y) j + 2z k$ from $O(0, 0, 0)$ to $P(3, 1, 2)$ along the line OP . 05
- Q.2
- a) Find a cosine series of period 2π to represent $\sin x$ in $0 \leq x \leq \pi$. 06
 - b) Find a, b, c if $\vec{F} = (axy + bz^3) i + (3x^2 - cz) j + (3xz^2 - y) k$ is irrotational. 06
 - c) Find the image of the circle $|z| = k$ where k is real under the bilinear transformation $w = \frac{5-4z}{4z-3}$. 08
- Q. 3
- a) Prove that $J_{\frac{1}{2}}(x) = \tan x \cdot J_{-\frac{1}{2}}(x)$. 06
 - b) Find the inverse Laplace transform of the following function by convolution theorem $\frac{(s+2)^2}{(s^2+4s+8)^2}$. 06
 - c) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in $(-l, l)$ where a is not an integer. 08
- Q. 4
- a) Find the angle between the normals to the surface $xy = z^2$ at the points $(1, 4, 2)$ and $(-3, -3, 3)$. 06
 - b) Prove that $x^2 J_n''(x) = (n^2 - n - x^2) J_n(x) + x J_{n+1}(x); n = 0, 1, 2, \dots$ 06

