

Total Marks: 80

Time Duration: 3Hr

N.B.: 1) Question no.1 is compulsory.

2) Attempt any three questions from Q.2to Q.6.

3) Figures to the right indicate full marks.

Maximum
Marks

- Q1. a)** Find the Laplace transform of $\cos 2t \sin t e^{-t}$. [5]
- b) Find the half-range sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. [5]
- c) Show that the function $f(z) = ze^z$ is analytic and find $f'(z)$ in terms of z . [5]
- b) Prove that $\nabla \left\{ \nabla \cdot \frac{\vec{r}}{r} \right\} = -\frac{2}{r^3} \vec{r}$. [5]
- Q2. a)** Find the inverse Z-transform of $F(z) = \frac{z}{(z-1)(z-2)}$ $|z| > 2$. [6]
- b) Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$. [6]
- c) Obtain Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & , -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & , 0 \leq x \leq \pi \end{cases}$, [8]
- deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
- Q3. a)** Find $L^{-1} \left[\frac{1}{s^2(s+a)^2} \right]$ using convolution theorem. [6]
- b) Show that the set of functions $\cos nx, n = 1, 2, 3 \dots$ is orthogonal on $[0, 2\pi]$. [6]
- c) Using Green's theorem evaluate $\int_C \left(\frac{1}{y} dx + \frac{1}{x} dy \right)$ where C is the boundary of the region defined by $x = 1, x = 4, y = 1$ and $y = \sqrt{x}$. [8]
- Q4. a)** Find Laplace transform of $f(t) = k \frac{t}{T}$ for $0 < t < T$ and $f(t) = f(t + T)$. [6]
- b) Show that $\vec{f} = (x^2 + xy^2) i + (y^2 + x^2y) j$ is irrotational and find its scalar potential. [6]
- c) Find half – range cosine series for $f(x) = x, 0 < x < 2$. Using Parseval's identity deduce that
- i) $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} - \frac{1}{5^4} + \dots$
- ii) $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$
- Q5.a)** Use divergence theorem to show that $\iint_S \nabla r^2 \cdot \vec{ds} = 6v$ where S is any closed surface enclosing a volume V. [6]
- b) Find the Z-transform of $f(k) = k\alpha^k, k \geq 0$. [6]
- c) i) Find $L^{-1} \left[\frac{(s+2)^2}{(s^2+4s+8)^2} \right]$ [8]
- ii) Find $L^{-1} [2 \tanh^{-1} s]$ [6]
- Q6.a)** Solve using Laplace transform $(D^2 - 3D + 2)y = 4e^{2t}$, with $y(0) = -3, y'(0) = 5$. [6]
- b) Find the bilinear transformation which maps the points 1, -i, 2 on z-plane onto 0, 2, -i respectively of w-plane. [6]
- c) Express the function $f(x) = \begin{cases} \sin x & , 0 < x \leq \pi \\ 0 & , x < 0, x > \pi \end{cases}$ as Fourier integral and deduce [8]
- that $\int_0^\infty \frac{\cos\left(\frac{w\pi}{2}\right)}{1-w^2} dw = \frac{\pi}{2}$.
