

(3 Hours)

[ Total marks : 80

- Note :-**
- 1) Question number 1 is **compulsory**.
  - 2) Attempt any **three** questions from the remaining **five** questions.
  - 3) **Figures** to the **right** indicate **full marks**.

- Q.1
- a) Evaluate  $\int_0^{\infty} e^{-2t} \sin^2 2t dt$ . 05
  - b) Find an analytic function  $f(z) = u + iv$  where  $u + v = e^x(\cos y + \sin y)$ . 05
  - c) Obtain Fourier series of  $x \cos x$  in  $(-\pi, \pi)$ . 05
  - d) Evaluate  $\int_C \bar{F} \cdot d\bar{r}$  where  $\bar{F} = x^2 i + xy j$  from  $(0, 0)$  to  $(1, 1)$  along the parabola  $y^2 = x$ . 05
- Q.2
- a) Find half-range cosine series for  $f(x) = e^x, 0 < x < 1$ . 06
  - b) Prove that  $\bar{F} = (x + 2y + az) i + (bx - 3y - z) j + (4x + cy + 2z) k$  is solenoidal and determine the constants  $a, b, c$  if  $\bar{F}$  is irrotational. 06
  - c) Prove that  $w = i \left( \frac{z-i}{z+i} \right)$  maps upper half of the  $z$  -plane into the interior of the unit circle in the  $w$  -plane. 08
- Q. 3
- a) Prove that  $J_n(x)$  is an even function if  $n$  is even integer and is an odd function if  $n$  is odd integer. 06
  - b) Find the inverse Laplace transform of  $\frac{s^2 + 2s + 3}{(s^2 + 2s + 5)(s^2 + 2s + 2)}$ . 06
  - c) Obtain the complex form of Fourier series for  $f(x) = e^{ax}$  in  $(0, a)$ . 08
- Q. 4
- a) Prove that  $\nabla f(r) = f'(r) \bar{r}$  and hence, find  $f$  if  $\nabla f = 2r^4 \bar{r}$ . 06
  - b) Prove that  $4J''_n(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$ . 06

- c)  
 (i) Find the Laplace transform of  $e^{4t} \sin^3 t$ . 04  
 (ii) Find the Laplace transform of  $t \sqrt{1 + \sin t}$ . 04
- Q. 5 a) Prove that  $\int x \cdot J_{\frac{2}{3}} \left(x^{\frac{3}{2}}\right) dx = -\frac{2}{3} x^{-\frac{1}{2}} J_{-\frac{1}{3}} \left(x^{\frac{3}{2}}\right)$ . 06  
 b) Find  $p$  if  $f(z) = r^2 \cos 2\theta + i r^2 \sin p\theta$  is analytic. 06  
 c) If  $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$  with period 2, show that 08  

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \cos(2n+1)\pi x.$$
- Q. 6 a) Show that the set of functions  $\cos nx$ ,  $n = 1, 2, 3, \dots$  is orthogonal 06  
 on  $(0, 2\pi)$ .  
 b) Use Stoke's theorem to evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where 06  
 $\vec{F} = (2x - y) i - yz^2 j - y^2 z k$  and  $S$  is the surface of hemisphere  
 $x^2 + y^2 + z^2 = a^2$  lying above the  $xy$  -plane.  
 c) Use Laplace transform to solve 08  
 $\frac{d^2 y}{dt^2} + y = t$  with  $y(0) = 1, y'(0) = 0$ .

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