

Duration – 3 Hours

Total Marks: 80

N.B 1) Question No. 1 is Compulsory.

2) Answer any three questions from remaining questions.

3) Figures to the right indicate full marks.

Q.1 a) Evaluate $\int_0^{\infty} y^4 e^{-y^6} dy$. 3

b) Find the circumference of a circle of radius r by using parametric equations of the circle $x = r \cos \theta, y = r \sin \theta$. 3

c) Solve $(D^2 + D - 6)y = e^{4x}$. 3

d) Evaluate $\int_0^1 \int_{x^2}^x xy(x^2 + y^2) dy dx$. 3

e) Solve $(\tan y + x)dx + (x \sec^2 y - 3y)dy = 0$. 4

f) Solve $\frac{dy}{dx} = 1 + xy$ with initial condition $x_0 = 0, y_0 = 0.2$ by Euler's method. Find the approximate value of y at $x = 0.4$ with $h = 0.1$. 4

Q.2 a) Solve $(D^2 - 4D + 3)y = e^x \cos 2x + x^2$. 6

b) Show that $\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2} \log(1+a)$. 6

c) Change the order of integration and evaluate $\int_0^2 \int_{\frac{x^2}{2}}^{4-x} xy dy dx$. 8

Q.3 a) Evaluate $\iiint x^2 yz dx dy dz$ throughout the volume bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. 6

b) Find the mass of lamina of a cardioid $r = a(1 + \cos \theta)$. If the density at any point varies as the square of its distance from its axis of symmetry. 6

c) Solve $(3x + 2)^2 \frac{d^2 y}{dx^2} + 5(3x + 2) \frac{dy}{dx} - 3y = x^2 + x + 1$. 8

Q.4 a) Find by double integration the area common to the circles $r = 2\cos\theta$ and $r = 2\sin\theta$. **6**

b) Solve $\sin 2x \frac{dy}{dx} = y + \tan x$. **6**

c) Solve $\frac{dy}{dx} = 3x + y^2$ with initial conditions $y_0 = 1$, $x_0 = 0$ at $x=0.2$ in steps of $h=0.1$ by Runge Kutta method of fourth order. **8**

Q.5 a) Evaluate $\int_0^1 x^5 \sin^{-1} x \, dx$ and find the value of $\beta \left(\frac{7}{2}, \frac{1}{2} \right)$. **6**

b) The differential equation of a moving body opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2 where x and v are the displacement and velocity of the particle at that time is given by $v \frac{dv}{dx} = -cx - bv^2$. Find the velocity of the particle in terms of x , if it starts from the rest. **6**

c) Evaluate $\int_0^6 \frac{dx}{1+4x}$ by using i) Trapezoidal ii) Simpsons (1/3)rd and iii) Simpsons (3/8)th rule. **8**

Q.6 a) Find the volume of the region that lies under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines $y = x$, $x = 0$ and $x + y = 2$ in the xy plane. **6**

b) Change to polar coordinates and evaluate $\iint y^2 dx dy$ Over the area outside $x^2 + y^2 - ax = 0$ and inside $x^2 + y^2 - 2ax = 0$. **6**

c) Solve by method of variation of parameters $\frac{d^2y}{dx^2} + y = \frac{1}{1+\sin x}$. **8**
