(3 hours) **Total Marks: 80**

Note:
1. Question no.1 is compulsory
2. Answer any three from remaining

1. a. Show that $\text{sech}^{-1}(\sin\theta) = \log\cot\left(\frac{\theta}{2}\right)$ (3)

b. Show that the matrix $A = \frac{1}{2}\begin{pmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{pmatrix}$ is unitary (3)

c. Evaluate $\lim_{x\to 0} \sin x \ \log x$ (3)

d. Find the nth derivative of $y = e^{ax} \cos^2 x \sin x$ (3)

e. If $x = r\cos\theta$ and $y = r\sin\theta$ prove that $JJ' = 1$ (4)

f. Using coding matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$ encode the message (4)

THE CROW FLIES AT MIDNIGHT

2. a. Find all values of $(1+i)^{\frac{1}{3}}$ and show that their continued product is (1 + i) (6)

b. Find the non singular matrices P & Q such that PAQ is in normal form where $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{pmatrix}$ (6)

c. Find max. and minimum values of $x^3 + 3x\,y^2 - 15\,x^2 - 15y^2 + 72x$ (8)

3. a. If $u = e^{xyz} f\left(\frac{xy}{z}\right)$ prove that $x\frac{\partial u}{\partial x} + z\,\frac{\partial u}{\partial z} = 2xyz\,u$ (6)

and $y\frac{\partial u}{\partial y} + z\,\frac{\partial u}{\partial z} = 2xyz\,u$ and hence show that

$$x\,\frac{\partial^2 u}{\partial z\partial x} = y\,\frac{\partial^2 u}{\partial z\partial y}$$

b. By using Regular falsi method solve 2x -3sin x -5 = 0 (6)

correct to three decimal places

c. If y = sin [log$(x^2 + 2x + 1)$] then prove that (8)

$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$

4. a. State and prove Eulers Theorem for three variables. (6)

b. By using De Moivres Theorem obtain tan 5θ in terms of (6)

tan θ and show that $1 - 10\,tan^2\left(\frac{\pi}{10}\right) + 5tan^4\left(\frac{\pi}{10}\right) = 0$

c. Investigate for what values of λ and μ the equations (8)

2x + 3y + 5z = 9

7x + 3y - 2z = 8

2x +3y + λz = μ have

(i) No solution
(ii) Unique solution
(iii) An infinite number of solution

5. a. Find nth derivative of $\frac{1}{x^2+a^2}$ (6)

b. If z = f(x,y) where x = $e^u + e^{-v}$, $y = e^{-u} - e^v$ then (6)

prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$

c. Solve by using Gauss Jacobi Iteration method (8)

2x + 12 y + z – 4 w = 13

13x + 5 y – 3z + w = 18

2x + y – 3z +9 w = 31

3x – 4y + 10 z + w = 29

6. a. If y = log [tan $(\frac{\pi}{4}+\frac{x}{2})$] Prove that (6)

(i) tan h $\frac{y}{2}$ = tan $\frac{x}{2}$

(ii) co shy cos x =1

b. If u = $sin^{-1}\left[\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right]^{1/2}$ prove that (6)

$$x^2\frac{\partial^2 u}{\partial^2 x}+ 2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial^2 y}=\frac{\tan u}{144}[tan^2u+13]$$

c.(i) Expand $2x^3 + 7x^2 + x - 6$ in powers of (4)

(x – 2) by using Taylors theorem.

(ii) Expand sec x by Maclaurins theorem considering upto x^4 term (4)

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